Key

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Graphing Simple Rational Functions

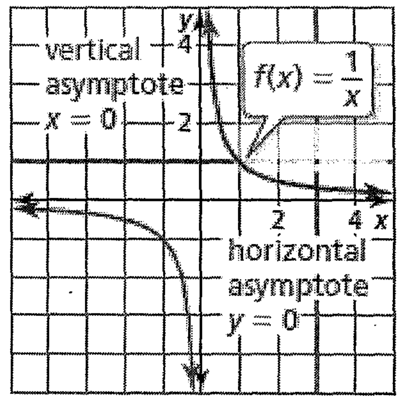
A **rational function** has the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$. The inverse variation function $f(x) = \frac{a}{x}$ is a rational function. The graph of this function when $a = 1$ is shown below.

Core Concept

Parent Function for Simple Rational Functions

The graph of the parent function $f(x) = \frac{1}{x}$ is a *hyperbola*, which consists of two symmetrical parts called branches. The domain and range are all nonzero real numbers.

Any function of the form $g(x) = \frac{a}{x}$ $(a \neq 0)$ has the same asymptotes, domain, and range as the function $f(x) = \frac{1}{x}$.



EXAMPLE 1 Graphing a Rational Function of the Form $y = \frac{a}{x}$

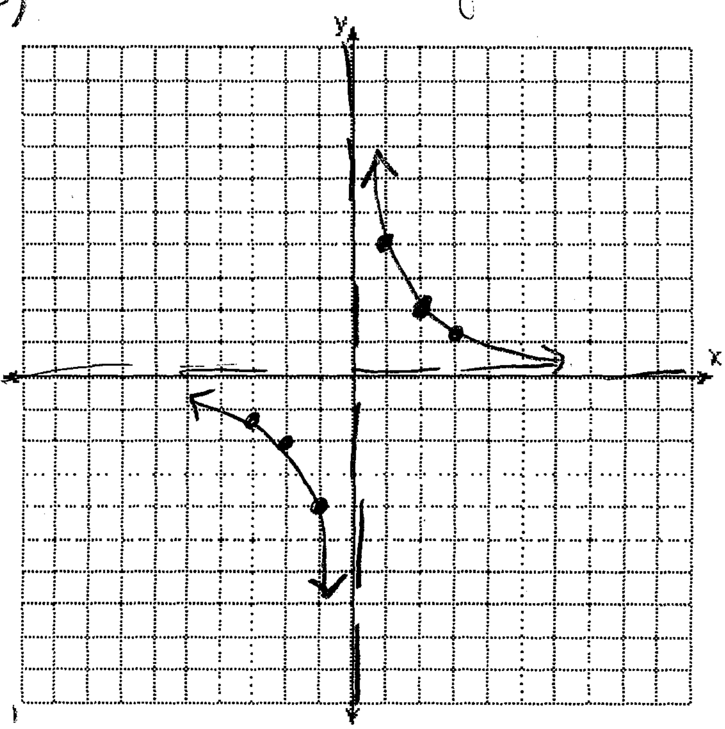
Graph $g(x) = \frac{4}{x}$. Compare the graph with the graph of $f(x) = \frac{1}{x}$. → can't be 0, undefined ∴ y can't be 0

$x, y \neq 0$ (asymptotes)

x	y
-3	-4/3
-2	-2
-1	-4
0	x
1	4
2	2
3	4/3

g(x) is a vert. stretch of f(x) by 4

$\left[4 \cdot \frac{1}{x} = \frac{4}{x}\right]$



Both in Quads. 1+3, same asy., D+R.

Translating Simple Rational Functions

Core Concept

Graphing Translations of Simple Rational Functions

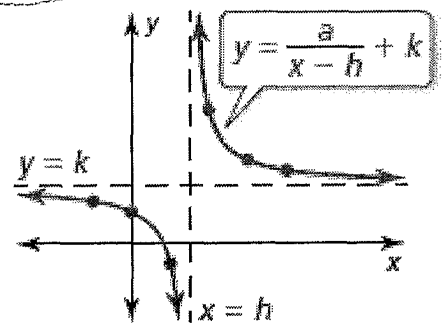
To graph a rational function of the form $y = \frac{a}{x-h} + k$, follow these steps:

whatever make undef

Step 1 Draw the asymptotes $x = h$ and $y = k$.

Step 2 Plot points to the left and to the right of the vertical asymptote.

Step 3 Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



EXAMPLE 2 Graphing a Translation of a Rational Function

Graph $g(x) = \frac{-4}{x+2} - 1$. State the domain and range.

undef.
$x+2=0$
$-2\ -2$
$x=-2$

VA: $x = -2$

HA: $y = -1$

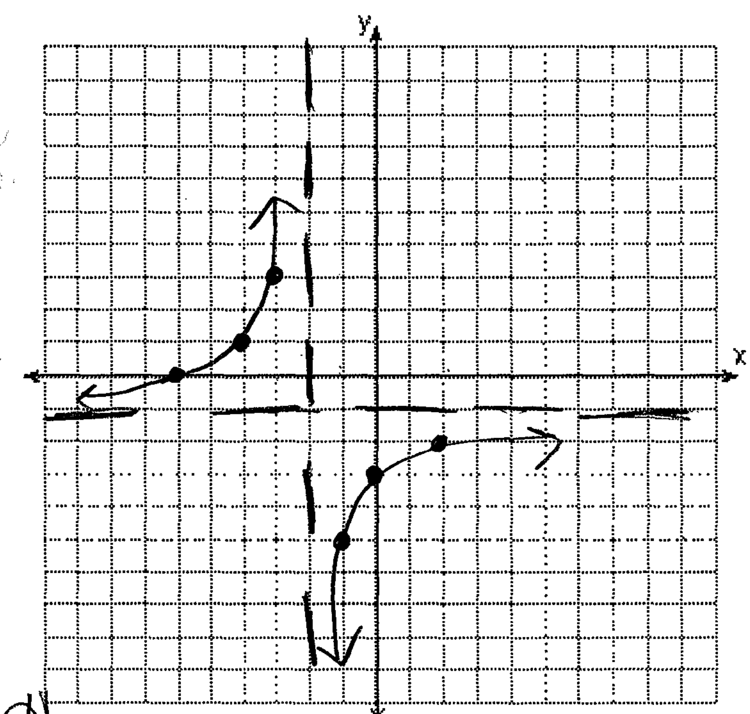
X	Y
-6	0
-5	
-4	1
-3	3
-1	-5
0	-3
1	
2	-2
3	

D: $\mathbb{R}, x \neq -2$

R: $\mathbb{R}, y \neq -1$

2 left, 1 down of $f(x) = \frac{-4}{x}$

reflect over y-axis



VA: $x = \frac{-d}{c}$
b/c undef. when denom = 0
HA: $\boxed{y = \frac{a}{c}\left(\frac{LC}{LC}\right)}$

EXAMPLE 3 **Graphing a Rational Function of the Form $y = \boxed{\frac{ax + b}{cx + d}}$**

Graph $f(x) = \frac{2x + 1}{1x - 3}$. State the domain and range.

$x \neq 3$ $y \neq 2$

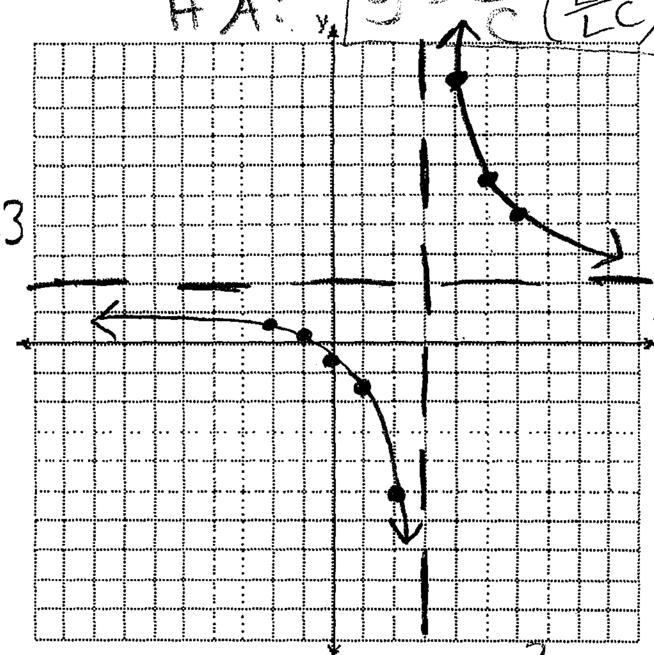
VA: $x - 3 = 0$
$+3\ \ +3$
$x = 3$

Also, $\frac{-(-3)}{1} = 3$

HA: $y = \frac{2}{1}$ $y = 2$

x	y
2	-5
1	-3/2
0	-1/3
-1	1/4
-2	3/5

x	y
4	9
5	11/2
6	13/3



$x - 3 \overline{)\ 2x + 1}$ quotient 2
$-(2x - 6)$
7

$f(x) = \frac{7}{x - 3} + 2$ 3 right, up 2

EXAMPLE 4 **Rewriting and Graphing a Rational Function**

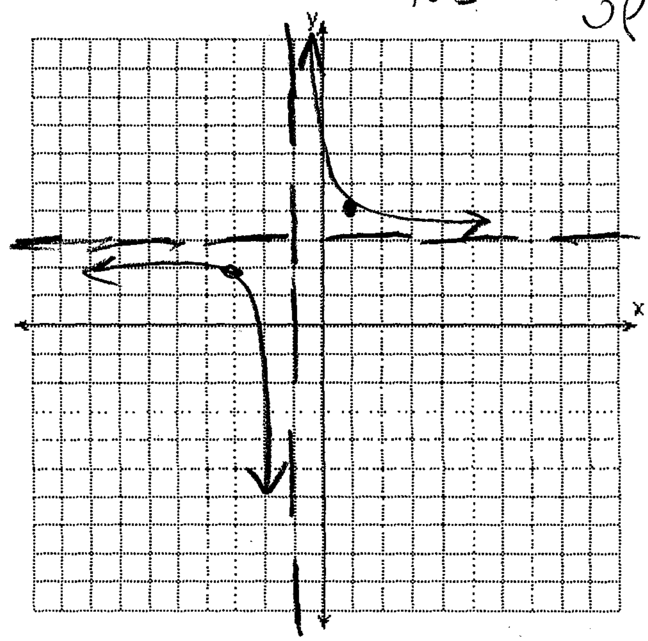
Rewrite $g(x) = \frac{3x + 5}{x + 1}$ in the form $g(x) = \frac{a}{x - h} + k$. Graph the function. Describe the graph of g as a transformation of the graph of $f(x) = \frac{a}{x}$.

long division: $x + 1 \overline{)\ 3x + 5}$ quotient 3
$-(3x + 3)$
2

$\frac{R}{\text{divisor}}$

$g(x) = 3 + \frac{2}{x + 1}$

OR $\frac{2}{x + 1} + 3$



left 1, up 3
of $f(x) = \frac{2}{x}$

VA: $x = -1$
HA: $y = 3$

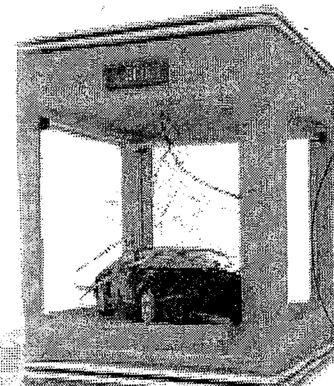
$\textcircled{*}$ $\frac{50m+1000}{m} = 90$

$90m = 50m + 1000$
$40m = 1000$
$m = 25$

EXAMPLE 5 Model g with Mathematics

A 3-D printer builds up layers f materials to make three-dimensional models. Each deposited layer bonds to the la er below it. A company decides to make small display models of engine components sing a 3-D printer. The printer costs $1000. The material for each model costs 0.

- Estimate how many models nust be printed for the average cost per model to f l to $90.
- What happens to the averag cost as more models are printed?



1, 1050
2, 550
3, 383
4, 300

SOLUTION printe = $1000
ma. for each model = $50

Avg. cost ⇒ $C = \frac{50m + 1000}{m}$ → # models

OR table (25, 90)

about 25 models

"Trace" in calc ≈ (25.106, 89.83)

VA: x=0

HA: y = 50, so as more models are printed, the avg. cost approaches $50 → $1000 fee becomes minimal!

What if printer cost $800?

- after 20 models are printed
- "C" still approaches $50

$C = \frac{50m + 800}{m} = 90$

$90m = 50m + 800$
$40m = 800$
$m = 20$

Key

Multiplying Rational Expressions

1) Sample Problem:

a) $\dfrac{2x^2+6x}{x+2} \bullet \dfrac{x^2-4}{x^2+3x}$	a) Original problem
b) $\dfrac{2x(x+3)}{x+2} \bullet \dfrac{(x+2)(x-2)}{x(x+3)}$	b) Factor all binomials and trinomials in the problem.
c) $\dfrac{2\cancel{x}\cancel{(x+3)}}{\cancel{x+2}} \bullet \dfrac{\cancel{(x+2)}(x-2)}{\cancel{x}\cancel{(x+3)}}$	c) Cancel out common factors vertically or diagonally only.
d) $2(x-2)$	d) Multiply across to get your final answer. Leave it in simplified form.

$\dfrac{x-3}{3-x} \to \dfrac{x-3}{-1(-3+x)}$

$= \dfrac{x-3}{-1(x-3)}$

Now you try:

2) $\dfrac{x^2+2x}{6x+18} \bullet \dfrac{x^2+5x+6}{x^2+4x+4}$

$\dfrac{x\cancel{(x+2)}}{6\cancel{(x+3)}} \cdot \dfrac{\cancel{(x+2)}\cancel{(x+3)}}{\cancel{(x+2)}\cancel{(x+2)}}$

$= \boxed{\dfrac{x}{6}}$

$(x \neq -3, -2)$

3) $\dfrac{x^2-12x+27}{x^2-81} \bullet \dfrac{x+9}{3-x}$

$\dfrac{\cancel{(x-9)}\cancel{(x-3)}}{\cancel{(x-9)}\cancel{(x+9)}} \cdot \dfrac{\cancel{x+9}}{\cancel{3-x}}$

-1

$= \boxed{-1}$

$(x \neq \pm 9, +3)$

DIVIDING RATIONAL EXPRESSIONS

1) Sample Problem:

a) $\dfrac{x^2-9}{4x} \div \dfrac{3x+9}{2x}$	a)Original problem
b) $\dfrac{x^2-9}{4x} \bullet \dfrac{2x}{3x+9}$	b)Keep the first fraction, change division to multiplication and flip the second fraction (KCF).
c) $\dfrac{(x-3)(x+3)}{4x} \bullet \dfrac{2x}{3(x+3)}$	c) Factor all binomials and trinomials in the problem.
d) $\dfrac{(x-3)\cancel{(x+3)}^{1}}{\cancel{4x}_{2}} \bullet \dfrac{\cancel{2x}^{1}}{3\cancel{(x+3)}_{1}}$	d) Cancel out common factors vertically or diagonally only.
e) $\dfrac{x-3}{6}$	e) Multiply across to get your final answer. Leave it in simplified form.

NOW YOU TRY:

2) $\dfrac{x^2-25}{x^2+7x} \div \dfrac{x^2+7x+10}{x^2+9x+14}$

$\dfrac{\cancel{(x+5)}(x-5)}{x\cancel{(x+7)}} \cdot \dfrac{\cancel{(x+7)}\cancel{(x+2)}}{\cancel{(x+5)}\cancel{(x+2)}}$

$= \boxed{\dfrac{x-5}{x}}$ $(x \neq 0, -7, -5, -2)$

3) $\dfrac{x^3-36x}{x^2+7x+6} \div \dfrac{6x^2-x^3}{x^2+x}$

$x(x^2-36)$

$\dfrac{\cancel{x}\cancel{(x+6)}^{1}\cancel{(x-6)}^{-1}}{\cancel{(x+6)}_{1}\cancel{(x+1)}_{1}} \cdot \dfrac{\cancel{x}\cancel{(x+1)}^{1}}{\cancel{x^2}\cancel{(6-x)}_{1}}$

$= \boxed{-1}$

$(x \neq \pm 6, -1, 0)$

RATIONAL EXPRESSIONS

ADDING OR SUBTRACTING WITH SAME DENOMINATORS

1.) Add, and express the sum in lowest terms:

$$\frac{x^2 - \cancel{5}}{2x^2} + \frac{\cancel{5} - 4x}{2x^2}$$

$$\frac{x^2 - 4x}{2x^2} = \frac{\cancel{x}(x-4)}{2x^{\cancel{2}}} = \frac{x-4}{2x}$$

$(x \neq 0)$

2.) Perform the indicated operation, and reduce to lowest terms:

$$\frac{7y-3}{y^2-9} - \frac{(y+15)}{y^2-9}$$

$$7y - 3 - y - 15 = 6y - 18 = \frac{6\cancel{(y-3)}}{(y+3)\cancel{(y-3)}}$$

$$= \frac{6}{y+3} \quad (y \neq \pm 3)$$

ex: ① $\frac{2}{x} + \frac{1}{4}$

② $\frac{6}{x} + \frac{5}{x^2}$

ADDING AND SUBT ACTING ALGEBRAIC FRACTIONS WITH DIFF RENT DENOMINATORS

Looking fo a Least Common Denominator

EXAMPLE	PROCEDURE
1.) $(x+1)\dfrac{x+2}{x^2-x}-\dfrac{6}{x^2-1}\cdot\dfrac{x}{x}$ $(x+1)x(x-1)\quad(x+1)(x-1)$ $x^2+3x+2-6x$ $x^2-3x+2=\dfrac{(x-2)(x-1)}{x(x+1)(x-1)}$ $(x\neq 0,\pm1)$	1) Factor denoms. 2) Multiply numer + denom. by whatever terms are needed to become the LCD 3) Combine numerators + create a single fraction 4) Factor 5) Reduce
2.) $\dfrac{-1\cdot 3y}{-1\cdot(2y-6)}+\dfrac{9}{6-2y}\rightarrow -2\quad +6$ $2(y-3)\qquad -2(y-3)$ $\dfrac{-3y+9}{-2(y-3)}=\dfrac{3\cancel{(y-3)}}{+2\cancel{(y-3)}}$ $=\dfrac{3}{2}$ $(y\neq 3)$	

COMPLEX FRACTIONS/RATIONAL EXPRESSIONS

1

Example	Procedure
$$\frac{\frac{x^2}{16} - 1 \cdot \frac{16}{16}}{\frac{x}{8} - \frac{1}{2}\left(\frac{4}{4}\right)} \qquad \frac{x^2}{16}-\frac{16}{16} \rightarrow \frac{x^2-16}{16} \qquad \frac{x}{8}-\frac{1}{2}\left(\frac{4}{4}\right) \rightarrow \frac{x-4}{8}$$ $$\frac{(x+4)\cancel{(x-4)}}{\cancel{16}\,2} \cdot \frac{\cancel{8}}{\cancel{x-4}}$$ $$= \frac{x+4}{2}$$ $(x \neq 4)$	1.) Change the numerator to a single fraction (do the subtraction.) 2.) Change the denominator to a single fraction (do the subtraction.) 3.) Rewrite the problem using ÷ . 4.) Keep, change, flip into a multiplication problem. 5.) Multiply. 6.) Simplify. -restrictions-

NOW YOU TRY:

2) $$\frac{\frac{x}{2}-\frac{8}{x}}{\frac{1}{4}-\frac{1}{x}}$$

$\frac{x}{x}\cdot\frac{x}{2} - \frac{8}{x}\cdot\frac{2}{2} \rightarrow \frac{x^2-16}{2x}$

$\frac{x}{x}\cdot\frac{1}{4} - \frac{1}{x}\cdot\frac{4}{4} \rightarrow \frac{x-4}{4x}$

$$\frac{(x+4)\cancel{(x-4)}}{\cancel{2x}} \cdot \frac{\cancel{4x}}{\cancel{x-4}}$$

$= 2(x+4)$

$(x \neq 0, 4)$

3) $$\frac{\frac{1}{x}-\frac{3}{x^2}+\frac{2}{x^3}}{\frac{1}{x}-\frac{4}{x^2}+\frac{4}{x^3}}$$

$$\frac{\frac{x^2-3x+2}{x^3}}{\frac{x^2-4x+4}{x^3}}$$

$$\frac{\cancel{(x-2)}(x-1)}{\cancel{x^3}} \cdot \frac{\cancel{x^3}}{\cancel{(x-2)}(x-2)}$$

$$\frac{x-1}{x-2}$$

$(x \neq 0, 2)$

Name ______________________________ Date ________

7.2 Practice A

In Exercises 1–3, graph the function. Compare the graph with the graph of $f(x) = \frac{1}{x}$.

1. $h(x) = \frac{2}{x}$
2. $g(x) = \frac{9}{x}$
3. $h(x) = \frac{-4}{x}$

In Exercises 4–15, graph the function. State the domain and range.

4. $f(x) = \frac{3}{x} + 2$
5. $y = \frac{5}{x} - 1$
6. $g(x) = \frac{4}{x - 3}$
7. $y = \frac{1}{x + 4}$
8. $h(x) = \frac{-1}{x + 3}$
9. $y = \frac{-4}{x - 5}$
10. $f(x) = \frac{x + 3}{x - 2}$
11. $y = \frac{x - 5}{x + 3}$
12. $g(x) = \frac{x + 4}{2x - 6}$
13. $y = \frac{5x + 2}{3x - 9}$
14. $h(x) = \frac{-2x + 3}{3x + 4}$
15. $y = \frac{8x - 1}{5x - 1}$

In Exercises 16–21, rewrite the function in the form $g(x) = \frac{a}{x - h} + k$. Graph the function. Describe the graph of g as a transformation of the graph of $f(x) = \frac{a}{x}$.

16. $g(x) = \frac{4x + 5}{x + 1}$
17. $g(x) = \frac{6x + 5}{x - 2}$
18. $g(x) = \frac{3x - 6}{x - 4}$
19. $g(x) = \frac{5x - 12}{x + 2}$
20. $g(x) = \frac{x + 15}{x - 5}$
21. $g(x) = \frac{x + 3}{x - 9}$

22. Your choir is taking a trip. The trip has an initial cost of $500, plus $150 for each student.

    a. Estimate how many students must go on the trip for the average cost per student to fall to $175.

    b. What happens to the average cost as more students go on the trip?

In Exercises 23–25, use a graphing calculator to graph the function. Then determine whether the function is *even*, *odd*, or *neither*.

23. $f(x) = \frac{5}{x^2 - 1}$
24. $g(x) = \frac{3x^2}{x^2 + 4}$
25. $h(x) = \frac{x^3}{2x^2 + x^4}$

Name____ Date__________

7.3 Practice A

In Exercises 1–6, simplify the expression, if possible.

1. $\dfrac{3x^2}{5x^2 + 2x}$

2. $\dfrac{6x^4 - x^3}{2x^4}$

3. $\dfrac{x^2 - 4x - 5}{x^2 - 7x + 10}$

4. $\dfrac{x^2 - 3x}{x^2 + 5x + 6}$

5. $\dfrac{x^2 - x - 2}{x^3 - 8}$

6. $\dfrac{x^2 - 3x - 4}{x^3 + 1}$

In Exercises 7–12, find the product.

7. $\dfrac{54x^4y^2}{y^4} \bullet \dfrac{x^3y^2}{9x^5y^3}$

8. $\dfrac{x^3(x + 2)}{x - 1} \bullet \dfrac{(x - 1)(x - 3)}{x^4}$

9. $\dfrac{x^2(x - 5)}{x + 7} \bullet \dfrac{(x + 7)(x - 1)}{4x^2}$

10. $\dfrac{x^2 - 5x}{x + 3} \bullet \dfrac{x^2 + 4x + 3}{x}$

11. $\dfrac{x^2 + 3x}{x - 2} \bullet \dfrac{x^2 - 5x + 6}{4x}$

12. $\dfrac{x^2 - 4x - 5}{x^2 + 6x + 9} \bullet \dfrac{2x^2 + 6x}{x^2 + 3x + 2}$

13. Compare the function $f(x) = \dfrac{(4x + 1)(x - 5)}{(4x + 1)}$ to the function $g(x) = x - 5$.

In Exercises 14–17, find the quotient.

14. $\dfrac{28x^4y}{y^7} \div \dfrac{y^9}{2x^5}$

15. $\dfrac{x^2 - x - 6}{3x^4 + 6x^3} \div \dfrac{x - 3}{6x^3}$

16. $\dfrac{4x^2 + 12x}{x^2 + 2x - 3} \div \dfrac{4x}{5x - 5}$

17. $\dfrac{x^2 + 5x - 14}{x + 3} \div (x^2 - 4x + 4)$

18. Manufacturers often package products in a way that uses the least amount of material. One measure of the efficiency of a package is the ratio of its surface area to its volume. The smaller the ratio, the more efficient the packaging. A company makes a cylindrical can to hold popcorn. The company is designing a new can with the same height h and twice the radius r of the old can.

    a. Write an expression for the efficiency ratio $\dfrac{S}{V}$, where S is the surface area of the can and V is the volume of the can.

    b. Find the efficiency ratio for each can.

    c. Did the company make a good decision by creating the new can? Explain.

Name ____ Date __________

7.4 Practice A

In Exercises 1–3, find the sum or difference.

1. $\frac{12}{5x} + \frac{3}{5x}$ **2.** [illegible] $- \frac{3}{9x^2}$ **3.** $\frac{7}{x-2} - \frac{3x}{x-2}$

In Exercises 4–7, find the least common multiple of the expressions.

4. $3x^2, 6x - 18$ **5.** $5x, 5x(x + 2)$

6. $x^2 - 9, x + 3$ **7.** $x^2 - 3x - 10, x + 2$

8. Describe and correct the error in finding the sum.

$$\times \quad \frac{x}{x+3} - \frac{2}{x-1} = \frac{x - 2}{(x+3)(x-1)}$$

In Exercises 9–12, find the sum or difference.

9. $\frac{7}{2x^2} - \frac{4}{3x}$ **10.** $\frac{2}{x-1} + \frac{4}{x+2}$

11. $\frac{6}{x+4} - \frac{5x}{x-3}$ **12.** $\frac{14}{x^2 + 7x - 18} + \frac{6}{x+9}$

In Exercises 13 and 14, tell whether the statement is *always*, *sometimes*, or *never* true. Explain.

13. The LCD of two rational functions is the sum of the denominators.

14. The LCD of two rational functions is equal to one of the denominators.

In Exercises 15–18, rewrite the function g in the form $g(x) = \frac{a}{x-h} + k$. Graph the function. Describe the graph of g as a transformation of the graph of $f(x) = \frac{a}{x}$.

15. $g(x) = \frac{4x - 5}{x - 2}$ **16.** $g(x) = \frac{5x + 3}{x + 4}$

17. $g(x) = \frac{10x}{x - 3}$ **18.** $g(x) = \frac{3x + 4}{x}$

Name ______________________________ Date __________

7.4 Practice A

In Exercises 1–3, find the sum or difference.

1. $\dfrac{12}{5x} + \dfrac{3}{5x}$

2. $\dfrac{x}{9x^2} - \dfrac{3}{9x^2}$

3. $\dfrac{7}{x-2} - \dfrac{3x}{x-2}$

In Exercises 4–7, find the least common multiple of the expressions.

4. $3x^2, 6x - 18$

5. $5x, 5x(x + 2)$

6. $x^2 - 9, x + 3$

7. $x^2 - 3x - 10, x + 2$

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In Exercises 9–12, find the sum or difference.

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10. $\dfrac{2}{x-1} + \dfrac{4}{x+2}$

11. $\dfrac{6}{x+4} - \dfrac{5x}{x-3}$

12. $\dfrac{14}{x^2 + 7x - 18} + \dfrac{6}{x+9}$

In Exercises 13 and 14, tell whether the statement is *always, sometimes,* or *never* true. Explain.

13. The LCD of two rational functions is the sum of the denominators.

14. The LCD of two rational functions is equal to one of the denominators.

In Exercises 15–18, rewrite the function g in the form $g(x) = \dfrac{a}{x-h} + k$. Graph the function. Describe the graph of g as a transformation of the graph of $f(x) = \dfrac{a}{x}$.

15. $g(x) = \dfrac{4x-5}{x-2}$

16. $g(x) = \dfrac{5x+3}{x+4}$

17. $g(x) = \dfrac{10x}{x-3}$

18. $g(x) = \dfrac{3x+4}{x}$

Name____ Date __________

7.5 Practice A

In Exercises 1–3, solve the equation by cross multiplying. Check your solution(s).

1. $\dfrac{3}{4x} = \dfrac{1}{x-2}$ **2.** $\dfrac{4}{x+2} = \dfrac{6}{x-2}$ **3.** $\dfrac{-3}{x+1} = \dfrac{x-5}{x-5}$

4. So far in baseball practice, you have pitched 47 strikes out of 61 pitches. Solve the equation $\dfrac{80}{100} = \dfrac{47+x}{61+x}$ to find the number x of consecutive strikes you need to pitch to raise your strike percentage to 80%.

In Exercises 5 and 6, identify the least common denominator of the equation.

5. $\dfrac{x}{x-2} + \dfrac{2}{x} = \dfrac{5}{x}$ **6.** $\dfrac{3x}{x+5} - \dfrac{8}{x} = \dfrac{2}{x}$

In Exercises 7–12, solve the equation by using the LCD. Check your solution(s).

7. $\dfrac{4}{3} + \dfrac{2}{x} = 4$ **8.** $\dfrac{5}{2x} + \dfrac{1}{4} = \dfrac{9}{2x}$

9. $\dfrac{x-2}{x-3} + 3 = \dfrac{2x}{x}$ **10.** $\dfrac{4}{x-5} + \dfrac{1}{x} = \dfrac{x-1}{x-5}$

11. $\dfrac{8}{x} + 3 = \dfrac{x+8}{x-4}$ **12.** $\dfrac{12}{x^2-2x} - \dfrac{3}{x-2} = \dfrac{3}{x}$

13. Describe and correct the error in the first step of solving the equation.

X

$$\frac{4}{x} + \frac{1}{2} = 1$$

$$2x \bullet \frac{4}{x} + 2x \bullet \frac{1}{2} = 1$$

14. You can clean the gutters of your house in 5 hours. Working together, you and your friend can clean the gutters in 3 hours. Let t be the time (in hours) your friend would take to clean the gutters when working alone. Write and solve an equation to find how long your friend would take to clean the gutters when working alone.

$\left(\textit{Hint:}\ (\text{Work done}) = (\text{Work rate}) \times (\text{Time})\right)$